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KEYNOTE

Equivalent-Circuit Modeling of Electrically-Very-Small Wireless Systems

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Abstract. This paper proposes an equivalent-circuit modeling technique for electrically-very-small wireless systems. The results obtained via the proposed method agree well with those via the induced electromotive force method, which is a very traditional analysis method for wire antennas.

1. Introduction

In recent years, wireless systems much smaller than the wavelength of electromagnetic waves (electrically-very-small) are widely used. Typical examples include near-field communication (NFC) systems, intrabody communication (IBC) systems [1], wireless power transfer (WPT) systems [2], and so on and so forth. Because such systems can be regarded as antenna problems as well as electric circuits, intense researches have been done by engineers in both the fields. However, sometimes their knowledges are separated and hard to integrate because the commonly used design analysis techniques are highly specialized in spite of the fact that they are based on the same physics, i.e. Maxwell's equations. This paper proposes an equivalent-circuit modeling technique for electrically-very-small wireless systems.

2. Theory

The proposed method is based on the method of moments (MoM), which is well-established in the field of computational electromagnetics [3]. If the space is uniform, isotropic, and non-dispersive, only the source of the electromagnetic wave is the current flowing on conductors. Therefore, all the antenna characteristics can be found once the current distributions are obtained. For this purpose, the MoM expands the current distributions $\boldsymbol{J}$ as follows:

$$\boldsymbol{J}(\boldsymbol{r}') = \sum_{n=1}^{N} I_n \boldsymbol{F}_n(\boldsymbol{r}'), \tag{1}$$

where $\boldsymbol{r}'$ is the source point, I_n is the unknown current coefficient, $\boldsymbol{F}_n(\boldsymbol{r}')$ is the known vector basis function, and N is the total number of them. The current coefficient I_n can be obtained by solving the following system of equations:

$$V_m = \sum_{n=1}^{N} Z_{mn} I_n\,, \quad m = 1, \dots, N, \tag{2}$$

where V_m is the voltage coefficient and Z_{mn} is the self-/mutual impedance. These quantities are defined as follows.

$$V_m = -\int \boldsymbol{F}_m(\boldsymbol{r}) \cdot \boldsymbol{E}^{\mathrm{src}}(\boldsymbol{r}) dV, \tag{3}$$

$$Z_{mn} = \gamma \frac{\zeta}{4\pi} \iint \boldsymbol{F}_m(\boldsymbol{r}) \cdot \boldsymbol{F}_n(\boldsymbol{r}') \frac{e^{-\gamma R}}{R} dV' dV + \frac{1}{\gamma} \frac{\zeta}{4\pi} \iint [\nabla \cdot \boldsymbol{F}_m(\boldsymbol{r})][\nabla' \cdot \boldsymbol{F}_n(\boldsymbol{r}')] \frac{e^{-\gamma R}}{R} dV' dV, \tag{4}$$

where $\gamma = s\sqrt{\varepsilon\mu}$ is the propagation constant and $\zeta = \sqrt{\mu/\varepsilon}$ is the wave impedance, supposing $s = j\omega$ is the complex angular frequency, ε is the permittivity, and μ is the permeability. In addition, $\boldsymbol{r}$ is the observation point, $R = |\boldsymbol{r} - \boldsymbol{r}'|$ is the distance between the source and the observation points, and the integration range is all over the space.

Now, substituting the Taylor expansion of the exponential function

$$e^{-\gamma R} = \sum_{i=0}^{\infty} \frac{(-\gamma R)^i}{i!}$$

into Eq. (4) and using an equality such that

$$\iint [\nabla \cdot \boldsymbol{F}_m(\boldsymbol{r})][\nabla' \cdot \boldsymbol{F}_n(\boldsymbol{r}')] dV' dV = 0,$$

we get the following expression:

$$Z_{mn} = \frac{1}{\gamma} \frac{\zeta}{4\pi} \iint [\nabla \cdot \boldsymbol{F}_m(\boldsymbol{r})][\nabla' \cdot \boldsymbol{F}_n(\boldsymbol{r}')] \frac{1}{R} dV' dV + \sum_{i=1}^{\infty} \gamma^i \left\{ \frac{(-1)^{i-1}\zeta}{(i-1)!\,4\pi} \iint \boldsymbol{F}_m(\boldsymbol{r}) \cdot \boldsymbol{F}_n(\boldsymbol{r}') R^{i-2} dV' dV + \frac{(-1)^{i+1}\zeta}{(i+1)!\,4\pi} \iint [\nabla \cdot \boldsymbol{F}_m(\boldsymbol{r})][\nabla' \cdot \boldsymbol{F}_n(\boldsymbol{r}')] R^i dV' dV \right\}. \tag{5}$$

Now, the term proportional to γ^i shall be denoted by $Z_{mn}^{(i)}$. The lowest order term $Z_{mn}^{(-1)}$ is identical to the impedance of the capacitance

$$C = 4\pi\varepsilon \left\{ \iint [\nabla \cdot \boldsymbol{F}_m(\boldsymbol{r})][\nabla' \cdot \boldsymbol{F}_n(\boldsymbol{r}')] \frac{1}{R} dV' dV \right\}^{-1}.$$

Besides, the term $Z_{mn}^{(1)}$, which is proportional to γ, corresponds to the impedance of the inductance

$$L = \frac{\mu}{4\pi} \iint \boldsymbol{F}_m(\boldsymbol{r}) \cdot \boldsymbol{F}_n(\boldsymbol{r}') \frac{1}{R} dV' dV + \frac{\mu}{8\pi} \iint [\nabla \cdot \boldsymbol{F}_m(\boldsymbol{r})][\nabla' \cdot \boldsymbol{F}_n(\boldsymbol{r}')] R dV' dV,$$

where the first term is equivalent to Neumann's formula.

The term $Z_{mn}^{(2)}$ can be simplified as follows:

$$Z_{mn}^{(2)} = -\gamma^2 \frac{\zeta}{6\pi} \left[\int \boldsymbol{F}_m(\boldsymbol{r}) dV \right] \cdot \left[\int \boldsymbol{F}_n(\boldsymbol{r}') dV' \right].$$

In particular, the self-impedance component $Z_{mm}^{(2)}$ is equivalent to the radiation resistance of the infinitesimal dipole with the length of

$$l = \left| \int \boldsymbol{F}_m(\boldsymbol{r}) dV \right|.$$

The proposed method is hereinafter called the impedance expansion method (IEM).

3. Example Problem

As shown in Fig. 1, a straight wire with the radius a and the length $3l$ has feeding ports 1 and 2 at the position of $z = l$ and $2l$, respectively. The current distributions along the wire are expanded by piecewise linear basis functions ($m = 1, 2$) defined as follows:

$$f_m(z) = \begin{cases} \dfrac{l - |ml - z|}{l}, & (m-1)l < z < (m+1)l \\ 0, & \text{elsewhere} \end{cases},$$

$$\frac{\partial f_m(z)}{\partial z} = \begin{cases} \dfrac{ml - z}{|ml - z|l}, & (m-1)l < z < (m+1)l \\ 0, & \text{elsewhere} \end{cases}.$$

They can be expressed as vector basis functions as

$$\boldsymbol{F}_m(\boldsymbol{r}) = \hat{z}\delta(x)\delta(y)f_m(z), \tag{6}$$

$$\nabla \cdot \boldsymbol{F}_m(\boldsymbol{r}) = \delta(x)\delta(y)\frac{\partial f_m(z)}{\partial z}. \tag{7}$$

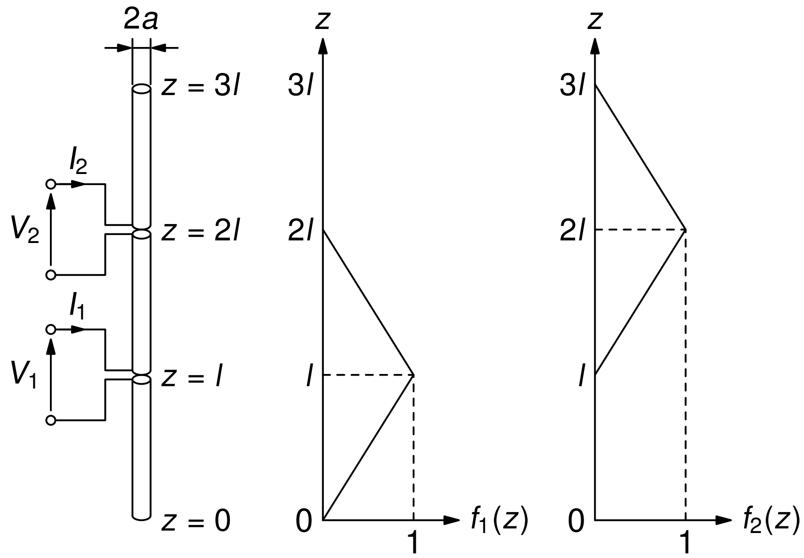

Fig. 1. Straight wire with two feeding ports and basis functions.

By substituting Eqs. (6) and (7) into Eq. (5) and ignoring the terms of $i \geq 3$, we get the following expressions for the self- and the mutual impedances:

$$\begin{aligned} Z_{mn} \simeq \frac{1}{\gamma}\frac{\zeta}{4\pi}\int_{(m-1)l}^{(m+1)l}\int_{(n-1)l}^{(n+1)l}\frac{\partial f_m(z)}{\partial z}\frac{\partial f_n(z')}{\partial z'}\frac{1}{R}dz'\,dz \\ +\gamma\left[\frac{\zeta}{4\pi}\int_{(m-1)l}^{(m+1)l}\int_{(n-1)l}^{(n+1)l}f_m(z)f_n(z')\frac{1}{R}dz'\,dz\right. \\ \left.+\frac{\zeta}{8\pi}\int_{(m-1)l}^{(m+1)l}\int_{(n-1)l}^{(n+1)l}\frac{\partial f_m(z)}{\partial z}\frac{\partial f_n(z')}{\partial z'}Rdz'\,dz\right] \\ -\gamma^2\frac{\zeta}{6\pi}\int_{(m-1)l}^{(m+1)l}\int_{(n-1)l}^{(n+1)l}f_m(z)f_n(z')dz'\,dz, \end{aligned} \tag{8}$$

where the distance between the source and the observation points is approximated as

$$R \simeq \sqrt{a^2 + (z - z')^2}.$$

The self- and the mutual impedances expressed by Eq. (8) can also be represented by the equivalent circuit shown in Fig. 2. The capacitances represent the impedance components proportional to γ^{-1} and can be calculated as follows:

$$p_{mn} = \frac{1}{4\pi\varepsilon}\int_{(m-1)l}^{ml}\int_{(n-1)l}^{nl}\frac{1}{R}dz'\,dz.$$

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{12} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}^{-1}$$

$$C_m = \sum_{n=1}^{3} c_{mn}, \quad C_{mn} = -c_{mn}$$

The inductances represent the impedance components proportional to γ and can be obtained as $L_{mn} = Z_{mn}^{(1)}/s$. The dependent voltage sources represent the impedance components proportional to γ^2 and can be obtained as follows:

$$\Delta V_1 = Z_{11}^{(2)} I_1 + Z_{12}^{(2)} I_2,$$
$$\Delta V_2 = Z_{21}^{(2)} I_1 + Z_{22}^{(2)} I_2.$$

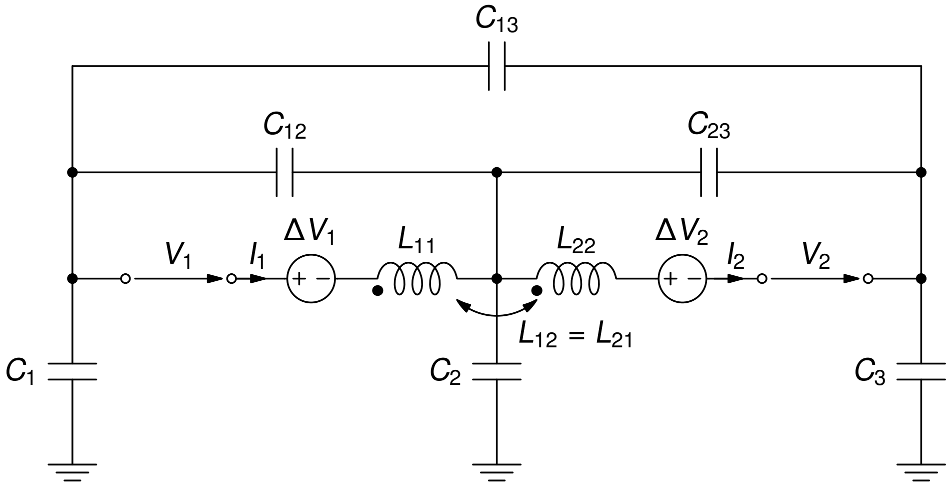

Fig. 2. Equivalent circuit derived via the IEM.

4. Numerical Examples

In this section, numerical results in the condition $a = 1.5$ mm and $l = 75$ mm are discussed. The circuit parameters are as follows:

$$C_1 = C_3 = 933.0174 \text{ [fF]}, \qquad C_2 = 799.3008 \text{ [fF]},$$
$$C_{12} = C_{23} = 216.6863 \text{ [fF]}, \qquad C_{13} = 45.40740 \text{ [fF]},$$
$$L_{11} = L_{22} = 36.60338 \text{ [nH]}, \quad L_{12} = L_{21} = 15.02211 \text{ [nH]},$$
$$\frac{Z_{11}^{(2)}}{s^2} = \frac{Z_{12}^{(2)}}{s^2} = \frac{Z_{21}^{(2)}}{s^2} = \frac{Z_{22}^{(2)}}{s^2} = -1.2508654\times10^{-18} \; [\Omega \cdot \text{s}^2].$$

Now, the frequency characteristics of the Y-parameters defined by

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1}$$

are discussed. Fig. 3 plots (a) the real and (b) the imaginary parts of Y_{11} and (c) the real and (d) the imaginary parts of Y_{21}. The dashed lines denoted by "EMF" indicate the results obtained via the induced electromotive force (EMF) method, which is a very traditional analysis method for wire antennas [4]. The results obtained via the IEM agree well with those via the EMF method. Therefore, the proposed circuit modeling method is valid.

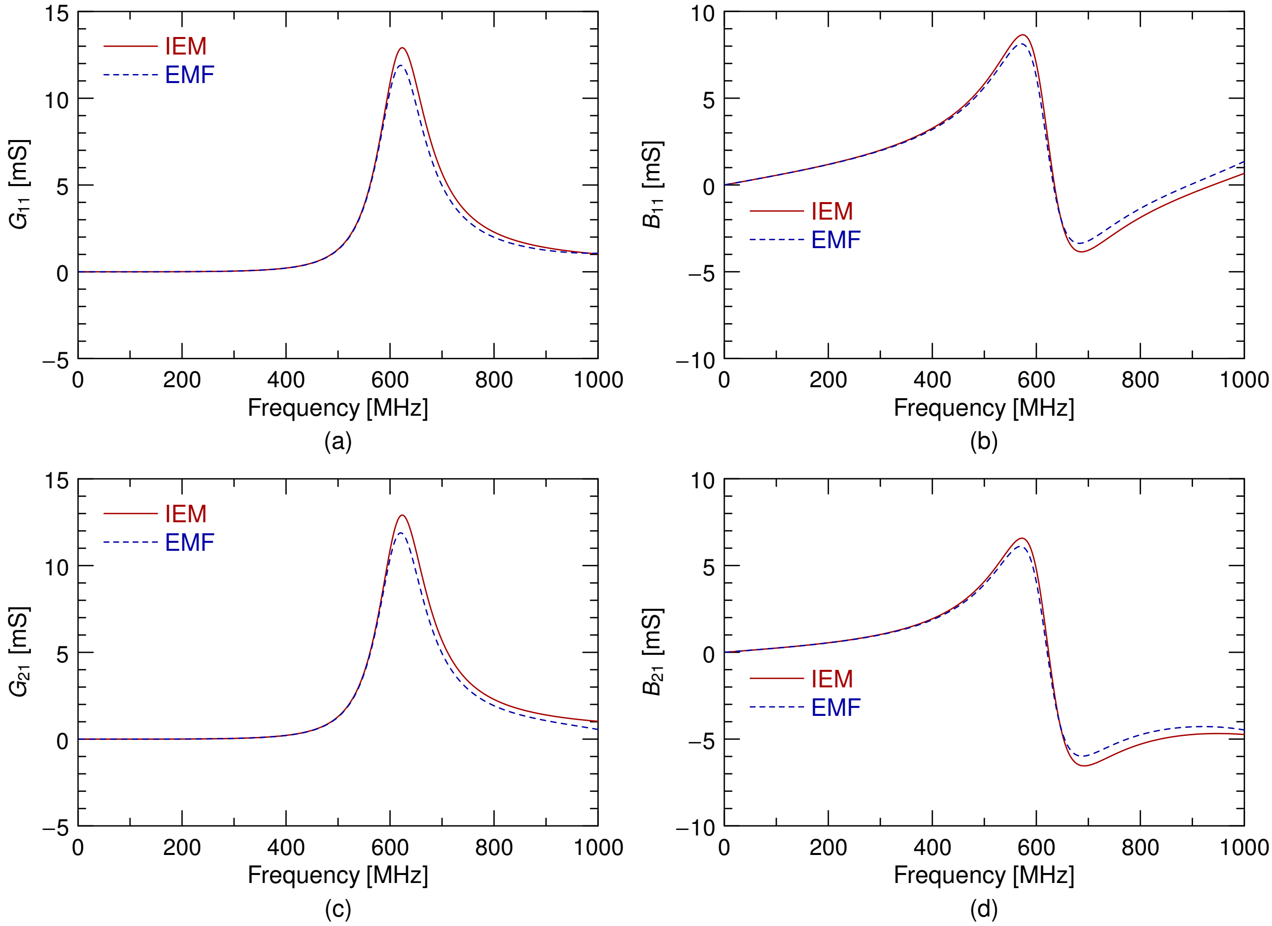

Fig. 3. Self-/mutual admittances: (a) G_{11}, (b) B_{11}, (c) G_{21}, and (d) B_{21}.

5. Conclusion

Basic theory and numerical examples of an equivalent-circuit modeling technique for electrically-very-small wireless systems were described. The proposed method is based on the method of moments. The results obtained via the proposed method agree well with those via the induced electromotive force method, which is a very traditional analysis method for wire antennas.

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