# Prediction of sound absorption coefficients from microscopic structure of poroelastic material by homogenization method

Takashi Yamamoto[1,a]

[1]Kogakuin University, Faculty of Mechanical Engineering, Tokyo, Japan

[a]<takashi_yamamoto@cc.kogakuin.ac.jp>

**Abstract.** Sound absorption coefficient in poroelastic media depends on material composed of and geometry in the microscale. Sound energy is generally absorbed by viscous dissipation in the vicinity of the boundary between solid and fluid phase, thermal dissipation to solid phase, and structural damping of elastic material which solid phase is composed of. Multi-scale model for sound-absorbing poroelastic media with periodic microscopic geometry has been recently proposed by one of the authors. In this method, and homogenization method based on the method of asymptotic expansions. Analysis in the microscopic scale is first performed by using unit cell of the periodic structure and macroscopic properties are derived. The properties are then applied to calculate macroscopic response such as sound absorption coefficient. In this paper we apply the proposed method to artificial poroelastic material with periodic pore of rectangular shape made by 3D printing technique and compare calculated sound absorption coefficient for normal incidence with measured one.

Keywords: Homogenization, Poroelastic material, Sound absorption coefficient, Additive manufacturing.

## 1. Introduction

Noise reduction to secure quietness in passenger compartments is one of the major issues in automotive engineering. One possible measure is to absorb sound by utilizing poroelastic media, e.g., floor carpets and dash insulators. In the design process of a vehicle, it is necessary to predict the macroscopic performance of poroelastic media, such as their sound absorption coefficients. Poroelastic material is composed of solid and fluid phases and the macroscopic performance of a poroelastic material is governed by the characteristics of each phase. Since those characteristics depend significantly on the microscopic geometry of the poroelastic material, predicting the macroscopic performance from the microscopic geometry would be essential for profound understanding of the physical behavior involved.

Macroscopic properties and governing equations can also be derived from the microscopic geometry by using the homogenization theory based on the method of asymptotic expansions, assuming that geometric periodicity exists on the micro-scale. Auliault et al. [1] considered a macroscopic description of rigid porous media saturated with an incompressible viscous fluid and derived a macroscopic permeability tensor that they verified experimentally. Terada et al. [2] studied the macroscopic characteristics of deformable poroelastic media saturated with an incompressible viscous fluid and presented numerical results to show the practical applicability of their approach. Lafarge et al. [3], Boutin et al. [4], and Lee [5] derived the macroscopic models of sound propagation through rigid porous media. Air contained in pores was modeled as a compressible viscous fluid, and the thermal dissipation from the fluid phase to the solid phase was also taken into account. Levy [6], and Burridge and Keller [7] derived the macroscopic governing equations of deformable poroelastic media saturated with a compressible viscous fluid. However, the thermal dissipation from the fluid phase into the solid phase was not taken into account and accordingly, the bulk modulus of the fluid phase was assumed to be constant. Although sound absorption in poroelastic media is a typical multiphysics phenomenon where the behavior of the elastic solid, the compressible viscous fluid and the fluid temperature must be all considered at the same time, the studies mentioned above deal with only some of the physics observed in sound-absorbing poroelastic media.

Therefore, in the study presented here, we propose a general and complete model that describes the macroscopic properties and the governing equations of sound-absorbing poroelastic media using the mathematical homogenization method. This model takes into account the motions of the elastic solid and compressible viscous fluid, and the distribution of temperature in the fluid. All physical variables mentioned in this study are shown in the frequency domain, assuming a harmonic regime with angular frequency $\omega$. Pressure, temperature and mass density in the fluid phase denote deviations from the equilibrium state with the pressure represented by $P^f$, the temperature $T^f$, and the mass density $\rho^f$.

## 2. Governing equations

The governing equations for a sound-absorbing poroelastic material on the microscopic scale are now considered. We assume that the solid phase is composed of linear elastic material and that the fluid phase is saturated with a compressible viscous fluid of viscosity $\mu^f$. The domain of the fluid phase is assumed to be connected throughout the material.

In the solid phase the linear equations of elasticity are given as follows:

$$-\rho^s\omega^2 u_i^s = \frac{\partial \sigma_{ij}^s}{\partial x_j}, \;\; \sigma_{ij}^s = c_{ijkl}^s \varepsilon_{kl}^s, \;\; \varepsilon_{kl}^s = \frac{1}{2}\left(\frac{\partial u_k^s}{\partial x_l} + \frac{\partial u_l^s}{\partial x_k}\right). \tag{1}$$

In the fluid phase the mass conservation law and the state equation of gas lead to

$$\rho^f \frac{\partial v_i^f}{\partial x_i} + j\omega\delta^f = 0, \;\; \frac{p^f}{P^f} = \frac{\delta^f}{\rho^f} + \frac{\tau^f}{T^f}, \tag{2}$$

where $\delta^f$ and $\tau^f$ are the perturbations of the mass density and the temperature, respectively.

Since infinitesimal harmonic motions are supposed in the fluid phase, the Navier-Stokes equation can be linearized and written as

$$\rho^f j\omega v_i^f = \frac{\partial \sigma_{ij}^f}{\partial x_j}, \;\; \sigma_{ij}^f = -p^f\delta_{ij} + 2\mu^f\dot{\varepsilon}_{ij}^f - \frac{2}{3}\mu^f\delta_{ij}\dot{\varepsilon}_{kk}^f, \;\; \dot{\varepsilon}_{ij}^f = \frac{1}{2}\left(\frac{\partial v_i^f}{\partial x_j} + \frac{\partial v_j^f}{\partial x_i}\right), \tag{3}$$

where $v_i^f$ is the velocity of the fluid phase.

When the specific heat capacity of the solid phase is much larger than that of the fluid phase, the solid phase can maintain isothermal conditions. Now let $q_i^f$, $C_v^f$, $R$, and $\kappa_{ij}^f$ be the thermal flux, the specific heat under constant volume, the gas constant, and the thermal conductivity, respectively. From the energy conservation law the set of governing equations for the temperature in the fluid phase is derived as

$$-\frac{\partial q_i^f}{\partial x_i} = j\omega\rho^f C_v^f\tau^f + (j\omega\rho^f R\tau^f - j\omega p^f), \;\; q_i^f = -\kappa_{ij}^f\frac{\partial \tau^f}{\partial x_j}. \tag{4}$$

When we consider the continuity of the velocity, the strain and the temperature, the conditions at the boundary $\Gamma^{sf}$ between the solid and fluid phases are given as follows:

$$j\omega u_i^s = v_i^f, \;\; \sigma_{ij}^s n_j^s + \sigma_{ij}^f n_j^f = 0, \;\; \tau^f = 0, \;\; \text{on} \;\; \Gamma^{sf}, \tag{5}$$

where $n_j^s$ and $n_j^f$ are the outward unit vectors normal to the boundary $\Gamma^{sf}$.

## 3. Boundary value problems for homogenization

Following Sanchez-Palencia [8], we assume a solution in the asymptotically expanded form for physical variables $u_i^s$, $v_i^f$, $p^f$, $\tau^f$, and $\delta^f$ such as $u_i^s = u_i^{s(0)}(\boldsymbol{x},\boldsymbol{y}) + \epsilon u_i^{s(1)}(\boldsymbol{x},\boldsymbol{y}) + \epsilon^2 u_i^{s(2)}(\boldsymbol{x},\boldsymbol{y}) + \cdots$, where all the functions on the right-hand side of the equations are $Y$-periodic, i.e., periodic in terms of $\boldsymbol{y}$ with the period of a unit cell $Y$. Then the forms of $u_i^s$, $v_i^f$, $p^f$, $\tau^f$, and $\delta^f$ are substituted into the governing equations.

From the linearity of the material, the displacement $u_i^{s(1)}$ is found to be linear with respect to stress and pressure, and can be written as $u_i^{s(1)} = -\chi_i^{kl}(\boldsymbol{y})\varepsilon_{kl}^{s(0)}(\boldsymbol{x}) - \eta_i(\boldsymbol{y})p^{f(0)}(\boldsymbol{x})$ where $\chi_i^{kl}(\boldsymbol{y})$ and $\eta_i(\boldsymbol{y})$ are $Y$-periodic characteristic functions. Setting $\varepsilon_{kl}^{s(0)}(\boldsymbol{x}) = 1$ and $p^{f(0)}(\boldsymbol{x}) = 0$ in the equilibrium equation leads to

$$\int_Y \left( c_{ijkl}^s - c_{ijpq}^s \frac{\partial \chi_p^{kl}(\boldsymbol{y})}{\partial y_q} \right) \frac{\partial \delta u_i^s}{\partial y_j} dY = 0. \tag{6}$$

Thus $\chi_i^{kl}(\boldsymbol{y})$ is obtained by solving this equation with a constraint condition $\int_Y \chi_i^{kl}(\boldsymbol{y})dY = 0$ to suppress rigid body modes. Similarly, setting $\varepsilon_{kl}^{s(0)}(\boldsymbol{x}) = 0$ and $p^{f(0)}(\boldsymbol{x}) = 1$ leads to

$$\int_Y c_{ijkl}^s \frac{\partial \eta_k(\boldsymbol{y})}{\partial y_l} \frac{\partial \delta u_i^s}{\partial y_j} dY = \int_{\Gamma^{sf}} \delta_{ij} n_j^s \delta u_i^s d\Gamma. \tag{7}$$

Thus $\eta_i(\boldsymbol{y})$ is also obtained by solving the equation with a constraint condition $\int_Y \eta_i(\boldsymbol{y})dY = 0$.

The fluid velocity relative to the solid phase $w^{f(0)}$ is defined by $w_i^{f(0)} = v_i^{f(0)} - j\omega u_i^{s(0)}$ at the order of $\epsilon^0$. Since $w_i^{f(0)}$ and $p^{f(1)}$ are found to be linear with respect to $W_i(\boldsymbol{x}) = w^{f(0)} + j\omega u_i^{s(0)}$, $w_i^{f(0)}$ and $p^{f(1)}$ can be expressed as $w_i^{f(0)} = \xi_i^k(\boldsymbol{y})W_k(\boldsymbol{x}),\ \ p^{f(1)} = \pi^k(\boldsymbol{y})W_k(\boldsymbol{x})$ where $\xi_i^k(\boldsymbol{y})$ and $\pi^k(\boldsymbol{y})$ are $Y$-periodic characteristic functions for the fluid velocity and pressure, respectively, and $\xi_i^k(\boldsymbol{y})$ must be 0 on $\Gamma^{sf}$. Setting $W_k(\boldsymbol{x})$ to 1, one can rewrite the equations as follows:

$$\int_Y \rho^f j\omega \xi_i^k(\boldsymbol{y})\delta w_i^f dY + \int_Y \mu^f \frac{\partial \xi_i^k(\boldsymbol{y})}{\partial y_j}\frac{\partial \delta w_i^f}{\partial y_j} dY + \int_Y \frac{1}{3}\mu^f \frac{\partial \xi_i^k(\boldsymbol{y})}{\partial y_i}\frac{\partial \delta w_i^f}{\partial y_i} dY - \int_Y \frac{\partial \xi_i^k(\boldsymbol{y})}{\partial y_i}\delta p^f dY - \int_Y \frac{\partial \delta w_i^f}{\partial y_i}\pi^k(\boldsymbol{y})dY = \int_Y \delta\xi_k^k(\boldsymbol{y})dY. \tag{8}$$

Therefore the characteristic functions $\xi_i^k(\boldsymbol{y})$ and $\pi^k(\boldsymbol{y})$ can be obtained by solving Eqs. (8) with a constraint condition $\int_Y \pi^k(\boldsymbol{y})dY = 0$.

Since the temperature $\tau^{f(0)}$ is linear with respect to the pressure $p^{f(0)}(\boldsymbol{x})$, $\tau^{f(0)}$ can be written as $\tau^{f(0)} = \frac{1}{\rho^f C_p^f}\zeta(\boldsymbol{y})p^{f(0)}(\boldsymbol{x})$ where $\zeta(\boldsymbol{y})$ is a $Y$-periodic characteristic function for the thermal field and satisfies the isothermal boundary condition, $\zeta(\boldsymbol{y}) = 0$. When $p^{f(0)}(\boldsymbol{x})$ is set to 1 in the thermal equilibrium equation, we obtain the following equation:

$$\int_Y \frac{1}{j\omega\rho^f C_p^f}\kappa_{ij}^f \frac{\partial \zeta(\boldsymbol{y})}{\partial y_j}\frac{\partial \delta\tau^f}{\partial y_i} dY + \int_Y \zeta(\boldsymbol{y})\delta\tau^f dY = \int_Y \delta\tau^f dY. \tag{9}$$

Thus the characteristic function $\zeta(\boldsymbol{y})$ for the temperature distribution in the fluid phase can be obtained by solving this boundary value problem.

## 4. Macroscopic governing equations

Averaging the equilibrium equations of the solid phase over a unit cell $Y$, and using $\langle v_i^{f(0)} \rangle = \langle w_i^{f(0)} \rangle + j\omega\phi u_i^{s(0)}$, $\langle \sigma_{ij}^{f(0)} \rangle = -\phi p^{f(0)}$, and $p^{f(0)} = -j\omega\psi^{f(0)}$, we can obtain the macroscopic governing equation for the solid phase as follows:

$$\frac{\partial \hat{\sigma}_{ij}^{s(0)}}{\partial x_j} + \bar{\rho}\omega^2 u_i^{s(0)} - \rho^f \omega^2 d_i^k u_k^{s(0)} - j\omega d_i^k \frac{\partial \psi^{f(0)}}{\partial x_k} + j\omega\phi \frac{\partial \psi^{f(0)}}{\partial x_i} + j\omega k_{ij}^H \frac{\partial \psi^{f(0)}}{\partial x_j} = 0, \tag{10}$$

where $\hat{\sigma}_{ij}^{s(0)} = c_{ijkl}^H \varepsilon_{kl}^{s(0)}$ is the stress of the solid phase uncoupled from the fluid phase, and $d_i^k$ is defined as $d_i^k = \rho^f j\omega \langle \xi_i^k(\boldsymbol{y}) \rangle$.

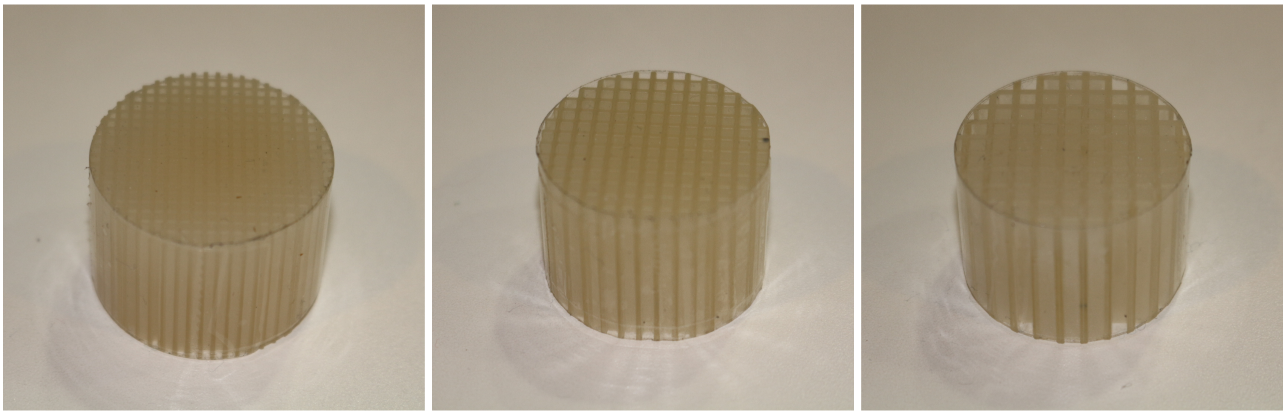
Averaging the equilibrium equations and the mass conservation law of the fluid phase over a unit cell $Y$, we can obtain the macroscopic governing equation for the fluid phase as follows:

$$\frac{d_i^k}{\rho^f} \frac{\partial^2 \psi^{f(0)}}{\partial x_k \partial x_i} + \omega^2 \left( \theta^f + \frac{\phi}{K^f} \right) \psi^{f(0)} - j\omega d_i^k \frac{\partial u_k^{s(0)}}{\partial x_i} + j\omega\phi \frac{\partial u_i^{s(0)}}{\partial x_i} + j\omega\theta^{s,pq} \varepsilon_{pq}^{s(0)} = 0. \tag{11}$$

where $\theta^{s,pq}$ and $\theta^f$ are defined respectively as $\theta^{s,pq} = \frac{1}{|Y|} \int_Y \frac{\partial \chi_k^{pq}(\boldsymbol{y})}{\partial y_k} dY, \;\; \theta^f = \frac{1}{|Y|} \int_Y \frac{\partial \eta_k(\boldsymbol{y})}{\partial y_k} dY.$

The coefficients on the macroscopic scale – $c_{ijkl}^H$, $k_{ij}^H$, $d_i^k$, $K^f$, $\theta^{s,ij}$, and $\theta^f$ – are fundamentally correlated with the microscopic structures of the poroelastic media.

## 5. Applications to artificial poroelastic material



(a) $w$=1.5mm,2$a$=1.0mm (b) $w$=2.2mm,2$a$=1.5mm (c) $w$=2.9mm,2$a$=2.0mm

Figure 1: Artificial poroelastic materials made by additive manufacturing.

We apply the method described above to artificial poroelastic material with periodic pores of square cross-section made by additive manufacturing technique, and compare calculated sound absorption coefficient for normal incidence with measured one to verify the proposed multiscale analysis.

Figure 5 shows artificial poroelastic materials made by Projet 3500 HD Max of 3D Systems. The size of square pores of Figs. 5 (a), (b) and (c) is 1.0mm, 1.5mm, and 2.0mm, respectively, and the wall thickness of the solid phase is 0.5 mm, 0.7 mm and 0.9 mm, respectively. The diameter and the thickness of these materials is 29 mm and 20 mm, respectively. Acryl is used for the material of the solid phase.

Sound absorption coefficients for normal incidence are measured by the acoustic tube of B&K Type 4206 and identified by the conventional two microphones method.

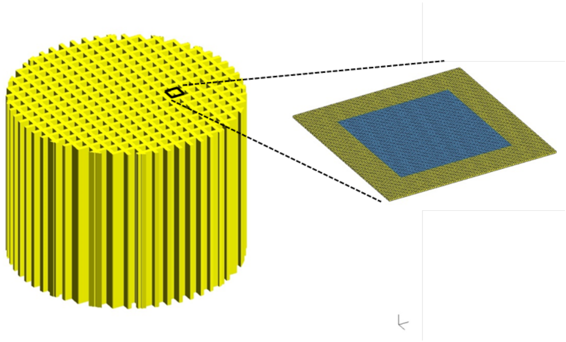
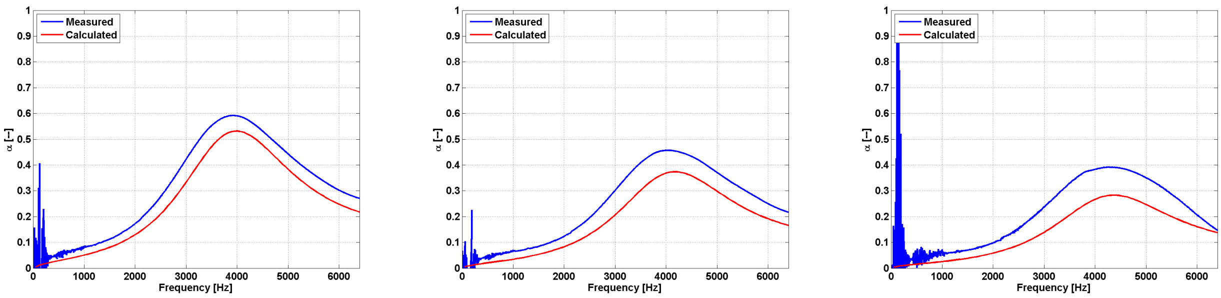


Figure 2: Finite element model in microscopic scale.




(a) $w$=1.5mm,2$a$=1.0mm (b) $w$=2.2mm,2$a$=1.5mm (c) $w$=2.9mm,2$a$=2.0mm

Figure 3: Comparisons with calculated and measured sound absorption coefficients.

Figure 5 shows finite element model of poroelastic material. Young's modulus, mass density and loss factor of acryl material are 1.46 GPa, 1020 kg/m$^3$ and 0.100, respectively. Mass density, speed of sound, viscosity, and themal conductivity of air are given as 1.19 kg/m$^3$, 345 m/s, 1.82 $\times$ 10$^{-5}$ Pa $\cdot$ s and 0.0257 W/m $\cdot$ K, respectively.

Figure 5 compares measured absorption coefficients and calculated one from 500 Hz to 6.4 kHz. Calculated sound absorption coefficients agree well with measured absorption coefficients.

## 6. Conclusions

A homogenized model for sound-absorbing poroelastic media was formulated based on the method of asymptotic expansions, taking into account viscous damping, thermal dissipation, and coupling effects between the solid and fluid phases. Several artificial poroelastic materials that have periodic cubic pores were made by 3D printer. Sound absorption coefficients calculated by the proposed approach agree well with those obtained by the measurement.

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