Proceedings of International Conference on Technology and Social Science 2017
Invited Paper

FEM for Narrow Slit Section Models with Damping of Air Viscosity

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Keywords: simulation, air viscosity, FEM, slit, damping

Abstract. This study deals with propagation of the sound waves in narrow slit section pathways. In very narrow pathways, the speed of sound propagation and the phase of sound waves change due to the air viscosity. We have developed a new finite element method (FEM) that includes the effects of air viscosity for modeling a narrow sound pathway. This method is developed as an extension of the existing finite element method for porous sound-absorbing materials. The numerical calculation results for several three-dimensional slit models using the proposed finite element method are validated against existing calculation methods. And relative error between the proposed method and the theoretical method was validated.

1. Introduction

The conventional acoustic analysis approach is used predominantly for relatively large structures or large equipment. For structures with a small volume, such as the occluded-ear simulator (IEC60318-4) for the measurement of insert-type earphones, very few methods of sound propagation analysis are available. This ear simulator has very narrow pathways to control the acoustic resistance; specifically, the speed of sound decreases and a phase delay occurs. Therefore, to perform accurate acoustic analysis of small devices, the effect of air viscosity should be considered. This effect is not considered in conventional acoustic analysis. In the present study, we developed a new FEM that considers the effects of air viscosity in narrow portions of the sound pathways in small devices. This method was developed as an extension of the acoustic FEM proposed by Yamaguchi [1, 2] for a porous sound-absorbing material. We attempted numerical analysis in the frequency domain using our acoustic solver, which utilizes the proposed FEM. For the numerical calculations, we used tube models with slit cross sections. Then we compared the results obtained by the proposed FEM with those obtained by theoretical analysis [3] and the conventional FEM.

2. Numerical Procedures

We have developed a new FEM that incorporates air viscosity at small amplitudes. Fig. 1 shows the direct Cartesian coordinate system and a constant strain element of a three-dimensional (3D) tetrahedron. Here, u_x, u_y, and u_z are the displacements in the x, y, and z directions at arbitrary points in the element. The strain energy $\tilde{U}$ can be expressed as follows:

$$\tilde{U} = \frac{1}{2} E \iiint_e \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} \right)^2 dxdydz \tag{1}$$

where E is the bulk modulus of elasticity of the air. The time derivative of the particle displacement is expressed as $\dot{u}$.

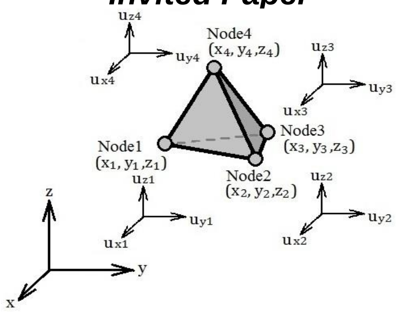

Fig. 1. Direct Cartesian coordinate system and a constant strain element

Therefore, the kinetic energy $\tilde{T}$ can be expressed as follows:

$$\tilde{T} = \frac{1}{2}\iiint_e \rho\{\dot{u}\}^T\{\dot{u}\}dxdydz \tag{2}$$

where ρ is the effective density of the element, and T represents a transposition. The viscosity energy $\tilde{D}$ of a viscous fluid can be expressed as follows:

$$\tilde{D} = \iiint_e \frac{1}{2}\{\overline{T}\}^T\{\Gamma\}dxdydz \tag{3}$$

where $\{\overline{T}\}$ is the stress vector attributable to viscosity. $\{\Gamma\}$ is the strain vector.

Next, we consider the formulation of the motion equation of an element for the acoustic analysis model that considers viscous damping. The potential energy $\tilde{V}$ can be expressed as follows:

$$\tilde{V} = \int_\Gamma \{u\}^T\{\overline{P}\}d\Gamma + \iiint_e \{u\}^T\{F\}dxdydz \tag{4}$$

where $\{\overline{P}\}$ is the surface force vector, $\{F\}$ is the body force vector, and $\int_\Gamma d\Gamma$ represents the integral of the element boundary. The total energy $\tilde{E}$ can be derived by using the following expression:

$$\tilde{E} = \tilde{U} + \tilde{D} - \tilde{T} - \tilde{V} \tag{5}$$

We can obtain the following discretized equation of an element by using Lagrange's equations:

$$\frac{d}{dt}\frac{\partial\tilde{T}}{\partial\dot{u}_{ei}} - \frac{\partial\tilde{T}}{\partial u_{ei}} + \frac{\partial\tilde{U}}{\partial u_{ei}} - \frac{\partial\tilde{V}}{\partial u_{ei}} + \frac{\partial\tilde{D}}{\partial u_{ei}} = 0 \tag{6}$$

where u_{ei} is the I th component of the nodal displacement vector $\{u_e\}$, and $\dot{u}_{ei}$ is the I th component of the nodal particle velocity vector $\{\dot{u}_e\}$. We can obtain the following discretized equation of an element by substituting Eq. (1)–(4) into Eq. (6).

$$-\omega^2[M_e]\{u_e\} + [K_e]\{u_e\} + j\omega[C_e]\{u_e\} = \{f_e\} \tag{7}$$

We use $\{\dot{u}_e\} = j\omega\{u_e\}$ in this equation because a periodic motion having angular frequency ω is assumed. $[M_e]$, $[K_e]$, $[C_e]$, and $\{f_e\}$ are the element mass matrix, element stiffness matrix, element viscosity matrix, and nodal force vector, respectively.

3. Calculation

3.1 Damping analysis by the three-dimensional finite element method

To verify our method, we carried out an acoustic damping analysis for slits using 3D FEM. As shown in Fig. 2, this model is a 1/4 solid model symmetrical about the *x*-*z* plane and the *x*-*y* plane. The width of the model was 2.0 mm, the height was 0.5 mm, and the length was 16.6 mm. This model used 3D tetrahedral elements having four nodes. There were 39 divided elements in the length (*x* direction), 10 layers in the height (*y* direction) and 3-10 layers in the width (*z* direction). Both ends of this model were closed. We selected the effective density ρ_R = 1.20 kg/m^3, the coefficient of viscosity $\mu = 1.82 \times 10^{-5}$ N•s/m^2, the real part of the complex volume elasticity $E_R = 1.40 \times 10^5$ Pa, and the sound propagation speed c = 340.0 m/s in air. As the boundary conditions, the particle displacements of all nodes on the outside in contact with surfaces were fixed, except for the plane of symmetry and the side walls which were not defined in the theoretical analysis. Fig. 3 shows the contours of the calculated particle displacements and the isosurface view of the model for the proposed finite element method, near the resonance conditions (10,200 Hz). As can be seen, the magnitude of the displacement of the particles changes significantly near the contact surface. However, the displacement becomes flatter with distance from the contact surface.

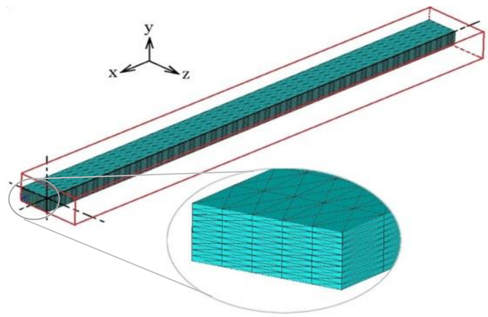

Fig. 2. Three-dimensional slit model for FEM

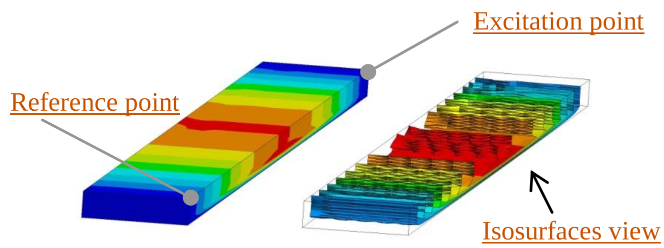

Fig. 3. The distribution of the particle displacement contour and the isosurface view(10,200Hz)

3.2 Damping analysis by theoretical analysis

We carried out theoretical analysis of the resonant response of the slit to verify the proposed FEM. The frequency response of the pressure can be generally expressed by the following general expression [4]:

$$P = -j\rho c v_0 e^{j\omega t} \frac{\cos k(x-l)}{\sin kl} \tag{8}$$

where ρ is density of the air, c is the speed of sound, l is the length of the tube, x is the position of a reference point, κ is ω/c, v_0 is the excitation velocity, and t is time. In this equation, we introduce the complex sound speed c^* and the complex effective density ρ_c^* to consider the attenuation due to the viscosity of the air. We replace the speed of sound and the density with the complex sound speed and the complex effective density as shown below.

$$c \;\Rightarrow c^* \;, \rho \;\Rightarrow\; \rho_c^* \tag{9}$$

Substitutions Eq. (9) into Eq. (8) , and the slit model is assumed infinite width. The effective density can be expressed as follow [3].

$$\rho_c^* = \frac{\rho}{1 - \dfrac{\tanh(s' j^{1/2})}{s' j^{1/2}}} \;, \; s' = \sqrt{\omega\rho\, a^2/\mu} \quad c^* = \sqrt{\kappa/\rho_c^*} = \sqrt{\gamma p_0/\rho_c^*} \tag{10}$$

where a is the distance between the contact surfaces, κ is the bulk modulus, p_0 is atmospheric pressure, and γ is the specific heat at constant volume.

3.3 Verification and comparison of the proposed method

We analyzed the frequency responses using the proposed FEM and compared the results with those of the above-described theoretical method that considers the viscosity and those of the conventional FEM that does not consider the attenuation. Fig. 4 shows the comparison of the analysis results for models of *a* = 0.8, 0.5, 0.25 mm. The condition of excitation was constant displacement excitation. From Fig. 4, we determined the effect of damping on the results obtained by the proposed FEM and the theoretical method. The conventional FEM does not show attenuation of the resonance peaks. As can be seen , when the slit is narrow, the resonance peak decreases. That is, the attenuation increases when the slit width is narrowed. In addition, it can be seen that the results obtained by the proposed FEM and the theoretical method over the entire frequency domain are approximately the same.

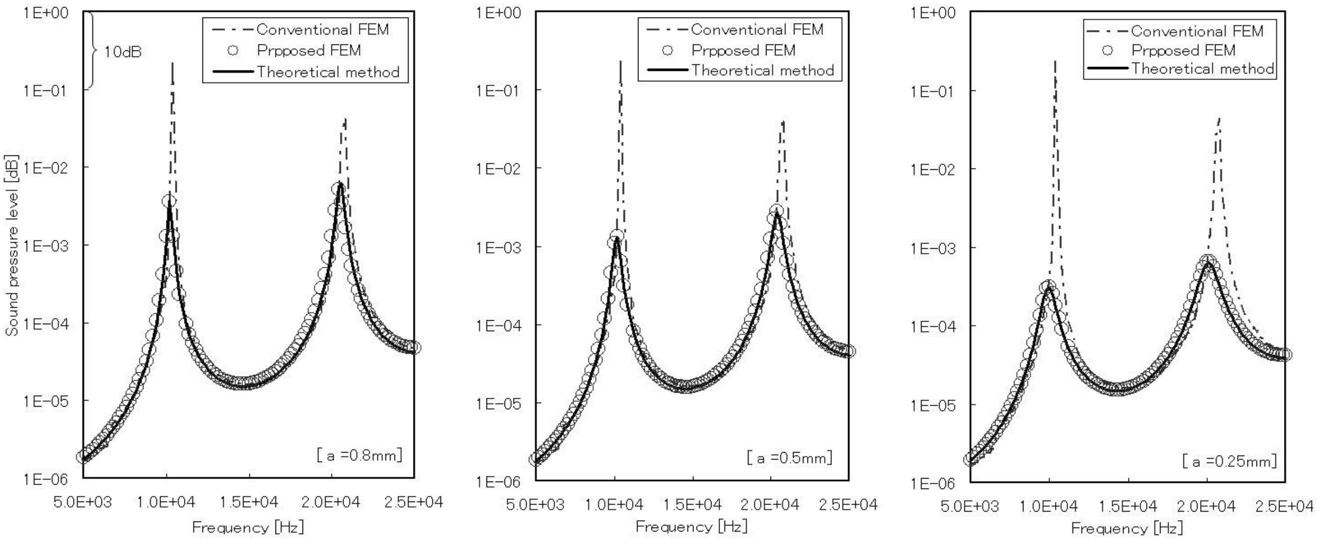

Fig. 4. Pressure versus frequency response for a slit model

4. Relative error between the proposed method and the theoretical method

Fig. 5 shows the relative error between the proposed method and the theoretical method for the case of a = 0.5 mm. Numbers (3, 5, 7, 9, and 10) are the number of element divisions in each of the x and y directions. Over all frequency bands from 5,000Hz to 25,000 Hz, the relative error is less than 1% for 5 divisions or more. And the relative error is less than 0.5% for 7 divisions or more. Further, the numerical solution can be confirmed to converge to the analytical solution with increasing number of component divisions.

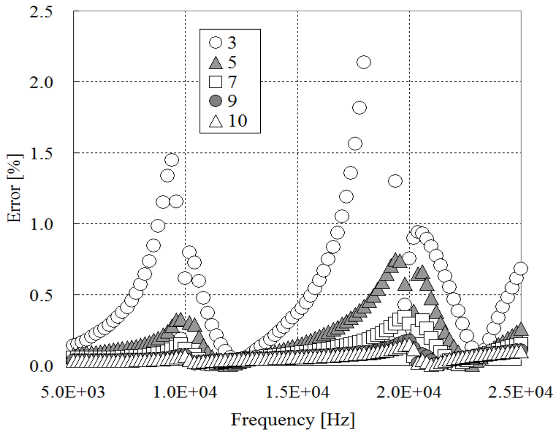

Fig. 5. Relative error between the proposed FEM and the theoretical method

5. Conclusion

We developed a new acoustic FEM that considers the effects of damping by the viscosity of air. We compared the sound pressure versus frequency characteristics obtained by the proposed method with those obtained by the theoretical method and the conventional acoustic FEM, which does not consider the effects of air viscosity in tubular models. The comparison showed that the obtained results are in good agreement. The proposed acoustic FEM was therefore confirmed to have good analytical accuracy. It was also confirmed that the numerical results of the proposed method converged to the theoretical solutions. It was therefore possible to confirm the effectiveness of the approach.

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